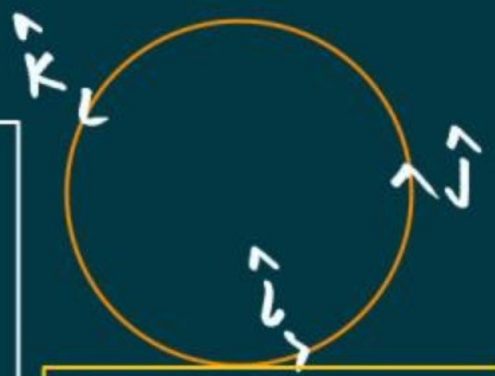
VECTOR (OR CROSS) PRODUCT OF TWO VECTORS

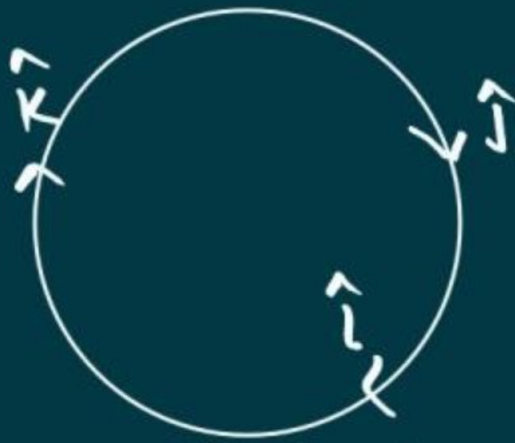
$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta \, \hat{n}$$

$\vec{b} \times \vec{a}$

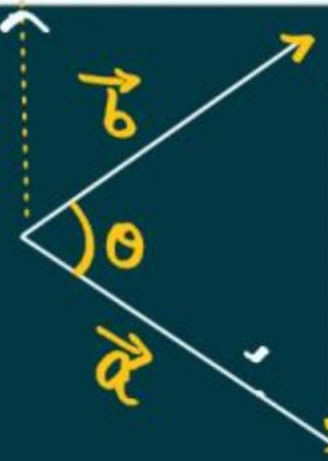
$\hat{i} \times \hat{j} = \hat{k}$	$\hat{i} \times \hat{i} = 0$
$\hat{j} \times \hat{k} = \hat{i}$	$\hat{j} \times \hat{j} = 0$
$\hat{k} \times \hat{i} = \hat{j}$	$\hat{k} \times \hat{k} = 0$



Anticlock wise



Clockwise



$\hat{j} \times \hat{i} = -\hat{k}$

$\hat{k} \times \hat{j} = -\hat{i}$

$\hat{i} \times \hat{k} = -\hat{j}$

Properties of vector (or cross) product

$\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$

$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

(i) $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

(ii) $(\vec{a} + \vec{b}) \times \vec{c} = \vec{a} \times \vec{c} + \vec{b} \times \vec{c}$

(iii) $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

$|\hat{n}| = 1$

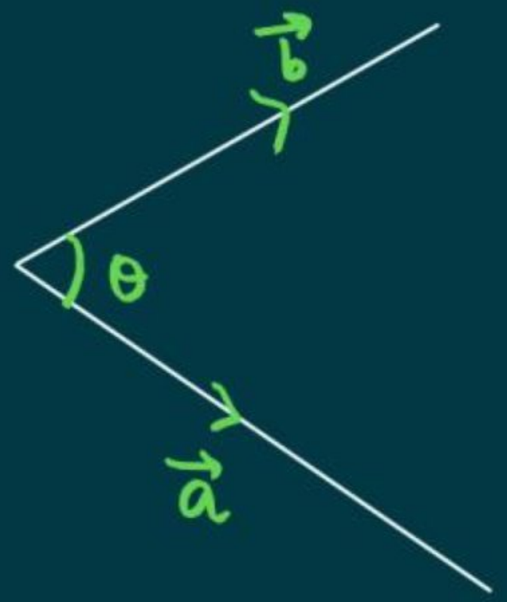
(iv) $\vec{a} \times \vec{b} = |\vec{a}|\,|\vec{b}| \sin\theta\ \hat{n}$

$|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \sin\theta$

$$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}|\,|\vec{b}|}$$

Angle between two vectors

$$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}|\,|\vec{b}|}$$



Lagrange's identity

if $\vec{a}, \vec{b}$ are any two vectors then $(\vec{a} \times \vec{b})^2 = |\vec{a}|^2 \, |\vec{b}|^2 - (\vec{a}.\vec{b})^2$

$|\vec{a} \times \vec{b}| = ab \sin\theta$

Sq. Both Side

$|\vec{a} \times \vec{b}|^2 = a^2 b^2 \sin^2\theta$

$|\vec{a}| = a$

$|\vec{b}| = b$

$|\vec{a} \times \vec{b}|^2 = a^2 b^2 (1 - \cos^2\theta)$

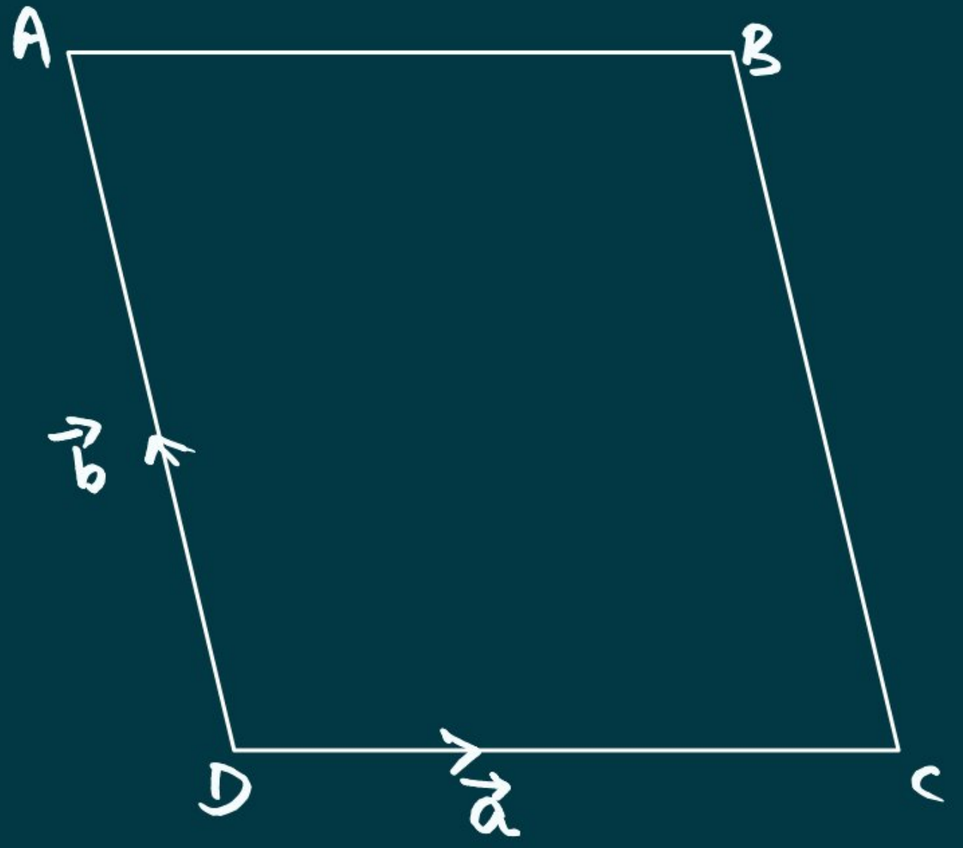
$= a^2 b^2 - a^2 b^2 \cos^2\theta$

$|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 - (ab\cos\theta)^2$

$\boxed{|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2}$

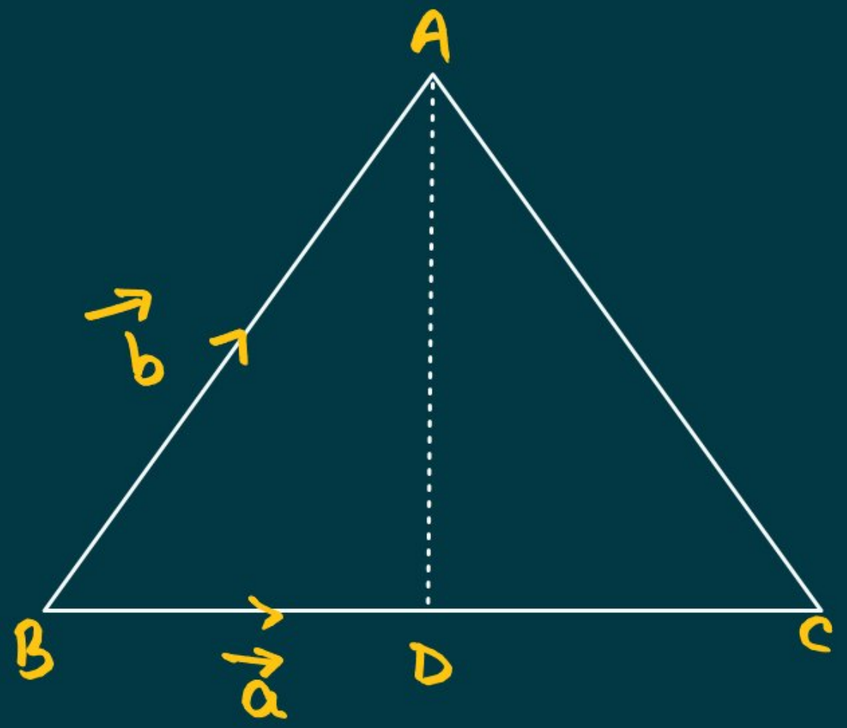
Area of a parallelogram

Area of a parallelogram $= |\vec{a} \times \vec{b}|$



Area of a triangle

Area of a Triangle $= \frac{1}{2}|\vec{a} \times \vec{b}|$



Question 1.

(i) If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$, then find $|\vec{a} \times \vec{b}|$. (NCERT)

(ii) Find the magnitude of $\vec{a}$, where $\vec{a} = (\hat{i} + 3\hat{j} - 2\hat{k}) \times (-\hat{i} + 3\hat{k})$.

(iii) If $\vec{a} = 3\hat{i} - 5\hat{j}$, $\vec{b} = 6\hat{i} + 3\hat{j}$ are two vectors and $\vec{c}$ is a vector such that $\vec{c} = \vec{a} \times \vec{b}$, then find $|\vec{a}| : |\vec{b}| : |\vec{c}|$.

(i) If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ $\quad \vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$ $\quad$ find $|\vec{a} \times \vec{b}|$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix}$$

$$= \hat{i}(-2-15) - \hat{j}(-4-9) + \hat{k}(10-3)$$

$$\vec{a} \times \vec{b} = -17\hat{i} + 13\hat{j} + 7\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(-17)^2 + (13)^2 + (7)^2} = \sqrt{289 + 169 + 49}$$

$\therefore$

$$\vec{a} = (\hat{i} + 3\hat{j} - 2\hat{k}) \times (-\hat{i} + 3\hat{k})$$

$$\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ -1 & 0 & 3 \end{vmatrix}$$

$$\hat{i}(9-0) - \hat{j}(3-2) + \hat{k}(0+3)$$

$$\vec{a} = 9\hat{i} - \hat{j} + 3\hat{k}$$

$$|\vec{a}| = \sqrt{(9)^2 + (-1)^2 + (3)^2}$$

$$= \sqrt{81 + 1 + 9} = \sqrt{91}$$

$\vec{a} = 3\hat{i} - 5\hat{j}$ $\quad \vec{b} = 6\hat{i} + 3\hat{j}$ $\quad \vec{c} = \vec{a} \times \vec{b}$

$$\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -5 & 0 \\ 6 & 3 & 0 \end{vmatrix}$$

$\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$

$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

$$\vec{c} = \hat{i}(0) - \hat{j}(0) + \hat{k}(9 + 30) = 39\hat{k}$$

$$|\vec{c}| = \sqrt{(39)^2} = 39$$

$$|\vec{a}| = \sqrt{(3)^2 + (-5)^2 + (0)^2}$$

$$= \sqrt{9 + 25} = \sqrt{34}$$

$$|\vec{b}| = \sqrt{(6)^2 + (3)^2 + (0)^2} = \sqrt{36 + 9 + 0} = \sqrt{45}$$ Ans

$$|\vec{a}| : |\vec{b}| : |\vec{c}| = \sqrt{34} : \sqrt{45} : 39$$

Question 2.

(i) Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively when $|\vec{a} \times \vec{b}| = \sqrt{3}$.

(ii) If vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3, |\vec{b}| = 2/3$ and $\vec{a} \times \vec{b}$ is a unit vector then find the angle between $\vec{a}$ and $\vec{b}$.

(iii) If $|\vec{a}| = 2$, $|\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, find the angle between $\vec{a}$ and $\vec{b}$.

(i) $|\vec{a}| = 1 \quad |\vec{b}| = 2$

$|\vec{a} \times \vec{b}| = \sqrt{3}$

$$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$$

$$\sin\theta = \frac{\sqrt{3}}{2 \times 1} = \frac{\sqrt{3}}{2}$$

$$\sin\theta = \frac{\sqrt{3}}{2}$$

$$\theta = 60^\circ$$

(ii) $|\vec{a}| = 3 \quad |\vec{b}| = 2/3 \quad |\vec{a} \times \vec{b}| = 1$

$$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$$

$$\sin\theta = \frac{1}{3 \times \frac{2}{3}} = \frac{1}{2}$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = 30^\circ$$

(iii) $|\vec{a}| = 2 \quad |\vec{b}| = 7 \quad \vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(3)^2 + (2)^2 + (6)^2}$$

$$= \sqrt{9 + 4 + 36}$$

$$|\vec{a} \times \vec{b}| = \sqrt{49} = 7$$

$$\sin\theta = \frac{7}{2 \times 7} = \frac{1}{2}$$

$$\sin\theta = \frac{1}{2}$$

$$\boxed{\theta = 30^\circ}$$

Question 3.

Find the angle between two vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$. (NCERT)

$$|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$$

$$|\vec{a}||\vec{b}| \sin\theta \, |\hat{n}| = |\vec{a}||\vec{b}| \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = 1$$

$$\tan\theta = 1$$

$$\boxed{\theta = \frac{\pi}{4}}$$

Question 4.

(i) Find $\vec{a} \cdot \vec{b}$ if $|\vec{a}| = 2$, $|\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$.

(ii) If $|\vec{a}| = 4$; $|\vec{b}| = 2$ and angle between $|\vec{a}|$ and $|\vec{b}|$ is $\frac{\pi}{3}$, find $(\vec{a} \times \vec{b})^2$.

(iii) If vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 1/2$, $|\vec{b}| = 4/\sqrt{3}$ and $|\vec{a} \times \vec{b}| = 1/\sqrt{3}$ then find $|\vec{a} \cdot \vec{b}|$.

$\vec{a} \cdot \vec{b} = ?$

Given $|\vec{a}| = 2 \quad |\vec{b}| = 5 \quad |\vec{a} \times \vec{b}| = 8$

$$|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$$

$$(8)^2 = (2)^2 (5)^2 - (\vec{a} \cdot \vec{b})^2$$

$$64 = 4 \times 25 - (\vec{a} \cdot \vec{b})^2$$

$$64 = 100 - (\vec{a} \cdot \vec{b})^2$$

$$(\vec{a} \cdot \vec{b})^2 = 100 - 64$$

$$(\vec{a} \cdot \vec{b})^2 = 36$$

$$\boxed{\vec{a} \cdot \vec{b} = \sqrt{36} = 6}$$